Proposed Algorithm of Tone Reservation PAPR Reduction in OFDM System

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Abstract:

Orthogonal frequency division multiplexing (OFDM) has become a popular modulation method in high-speed wireless communication systems due to its high data rates transmission capability and robustness against multipath fading effects. One of the major drawbacks of OFDM at the transmitter side is the high peak-to-average power ratio (PAPR) of the OFDM signal. In this paper, an algorithm is proposed to reduce the peak to average power ratio of OFDM signal with large number of sub-carriers. This algorithm is based on tone reservation method. The computer simulation tests shoe that the suggested algorithm reduces the PAPR to a factor of 5.25 dB and needs less number of iterations as compared with the traditional tone reservation algorithm.

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الخلاصة:

منظومة تقسيم التردد المتعامد (OFDM) أصبحت المنظومة الأكثر شيوعا بين أنظمة الاتصالات اللاسلكية ذات السرعة العالية وذلك لقدرتها على التعامل مع سرع الإرسال العالية جدا و تجنب مشاكل المسارات المتعددة. أحد العيوب الأساسية لمنظومة(OFDM) عند طرف الإرسال هو النسبة العالية لأعلى قدرة إلى معدل هذه القدرة (PAPR) في الإشارة المرسلة. في هذا البحث تم اقتراح تقنية لتقليل تلك النسبة في المنظومة ذات العدد الكبير من الترددات الحاملة الثانوية. التقنية المقترحة تعتمد أساسا على طريقة حفظ الترددات النغمية. أثبتت نتائج المحاكاة بالحاسبة أن التقنية المقترحة تقل (PAPR) إلى حدود (5.25 dB) مع احتياجها إلى عدد أقل من العمليات التكرارية إذا ماتم مقارنتها مع الطرق التقليدية.

1 OFDM System Model:-

Basically, OFDM transmitter divides the bandwidth into several orthogonal sub-carriers. Nevertheless, one of the main disadvantages of this modulation is its high peak to average power ratio requiring the use of linear high power amplifiers that are very power-inefficient. Figure 1, shows a baseband representation of an OFDM transmitter with cyclic prefix.

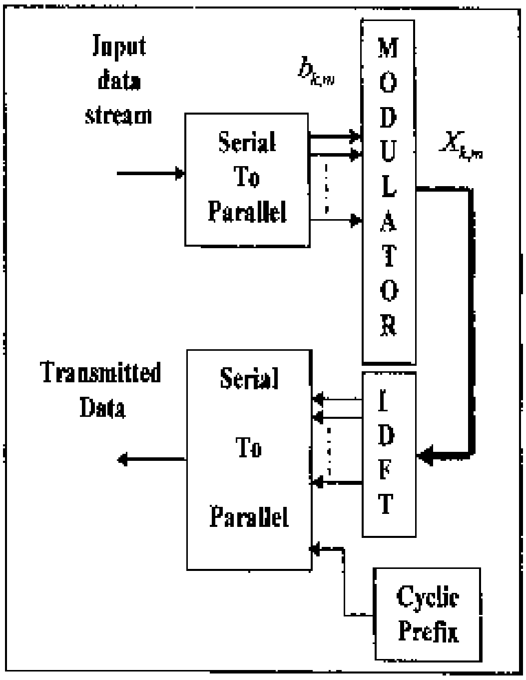

Figure 1: Digital multi-carrier transmission system applying OFDM.

Binary data stream of length L are divided to N blocks $b_{k,m}$ by a serial to parallel (S/P) converter, each block will be modulated onto the symbols $X_{k,m}$ (a complex number that corresponds to a signal constellation). Different modulation schemes like Quadrature Phase Shift Keying (QPSK) are used to modulate the data stream. Here, it is assumed that in all active carriers of OFDM $X_{k,m}$ the same complex-valued signal set χ or the same constellation size with variance σ_X^2 is used. The inactive carriers are set to zero in order to shape the power density spectrum of the transmit signal appropriately. The channel period T related to the symbol period of the modulation scheme used in each sub-carrier (T_s) is taken the form:

$$T_s = N \cdot T \qquad (1)$$

The vector $X_{k,m}$ comprising the carrier amplitudes associated with OFDM symbol interval m is transformed into time domain using IDFT. This has the form of the T-spaced discrete-time representation of the m-th block of the transmit signal [1,2]:

$$s_{n,m} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_{k,m} \cdot e^{j\frac{2\pi}{N}kn},$$

$$0 \le n \le N-1 \qquad (2)$$

where n is the discrete time index.

2 Theoretical Analysis of PAPR

Many literatures use the crest factor as a figure of merit in the analysis of the peak to average power ratio in OFDM systems. The crest factor of the discrete-time representation is defined as the ratio of the peak magnitude value to the square root of the average power of this signal. The T-spaced samples within the symbol period associated with OFDM symbol are directly obtained from the sequences s_m with elements $s_{n,m}$. Thus, the crest factor can be written as [3]:

$$\zeta = \frac{\max_{\forall m, 0 \le n \le N} |s_{n,m}|}{\sqrt{\varepsilon\{|s_{n,m}|^2\}}} \tag{3}$$

where $\varepsilon\{\}$ denotes the expected value. Note that the peak-to-average power ratio is simply the square of the crest factor. The mean-squared magnitude σ_x^2 of the sequence $s_{n,m}$ or variance for time domain OFDM symbol is calculated according to Parsevals's theorem, resulting as in [3].

$$\sigma_x^2 = \varepsilon\{|s_{n,m}|^2\} = \frac{\sqrt{N}}{N}\sigma_X^2$$

$$= \frac{\sigma_X^2}{\sqrt{N}} \tag{4}$$

where the σ_X^2 variance of frequency domain OFDM symbol. By substituting equation (4) in equation (3), the crest factor may be represented by:

$$\zeta = \frac{\sqrt{N} \cdot \max_{0 \le n < N} |s_{n,m}|}{\sqrt{\sigma_X^2}} \tag{5}$$

In this analysis, it is important to calculate the probability that the crest factor of an OFDM symbol exceeds a certain threshold. The Central Limit Theorem states that the probability density of the sum of a large number of independently distributed quantities approaches Gaussian probability density function, regardless of the form of distribution of the individual components [4, 5].

Regarding equation (2), the time-domain samples of the OFDM signal are the sum of N nonzero carrier amplitudes $X_{k,m}$ which can be interpreted as independent zero-mean random variables with variance σ_X^2. The multiplication with exponential complex factors does not affect the variance of the individual components. Due to the central limit theorem, and assuming that N is sufficiently large, $s_{n,m}$ is a zero-mean complex near Gaussian distributed random variable with variance in the above equation. Introducing the variable $u = |s_{n,m}|$, we obtain the Rayleigh distribution for the probability density function of the OFDM signal magnitude [6, 7].

$$p_u(u)=\frac{2u}{\sigma_x^2}\exp\left(-\frac{u^2}{\sigma_x^2}\right) \qquad (6)$$

The probability that the magnitude of one single signal sample does not exceed a certain amplitude threshold a_0 can therefore be calculated as [5]

$$\Pr\{|s|\le a_0\}=\int_0^{a_0} p_u(u)\,du$$
$$=1-\exp\left(-\frac{a_0^2}{\sigma_x^2}\right) \qquad (7)$$

Assuming $s_{n,m}$, to be statistically independent, the probability that the magnitude value of an entire OFDM symbol which does not exceed a certain magnitude threshold can be approximated by Cumulative Distribution Function (CDF) [6]

$$CDF=\Pr\{|s_{0,m}|\le a_0,\ldots,|s_{N_c-1,m}|\le a_0\}$$
$$=\Pr\{|s_{n,m}|\le a_0\}^N$$
$$=\left(1-\exp\left(-\frac{a_0^2}{\sigma_x^2}\right)\right)^N \qquad (8)$$

The Complementary Cumulative Distribution Function (CCDF) of the PAPR denotes the probability that the PAPR of an entire OFDM symbol that at least one magnitude exceeds a given threshold.

$$\Pr(PAPR>\zeta_0)=1-CDF$$
$$=1-\left(1-\exp\left(-\frac{a_0^2}{\sigma_x^2}\right)\right)^N \qquad (9)$$

In another words, the theoretical expression of the probability $P_\zeta(\zeta_0)$ that the crest factor of one OFDM symbol at time m exceeds a certain crest factor threshold $\zeta_0=\frac{a_0}{\sigma_x}$ follows from the above as

$$P_\zeta(\zeta_0)=\Pr\{\zeta>\zeta_0\}$$
$$=1-\left(1-\exp(-\zeta_0^2)\right)^N \qquad (10)$$

Due to the approximation, the probability of the occurrence of OFDM symbols having a crest factor ζ higher than a given threshold ζ_0 is merely a function of the IDFT length N used in the given OFDM transmitter [7].

3 State of Problem

Figure 2 illustrates the probability that crest factor of the OFDM signal exceed a certain value by evaluating the expression in equation (10) using a computer simulation test. In this test, it is assumed that the lengths of the inverse discrete Fourier transform are 16, 64 and 128. The input data has a complex value of ± 1 and then mapped using QPSK modulator.

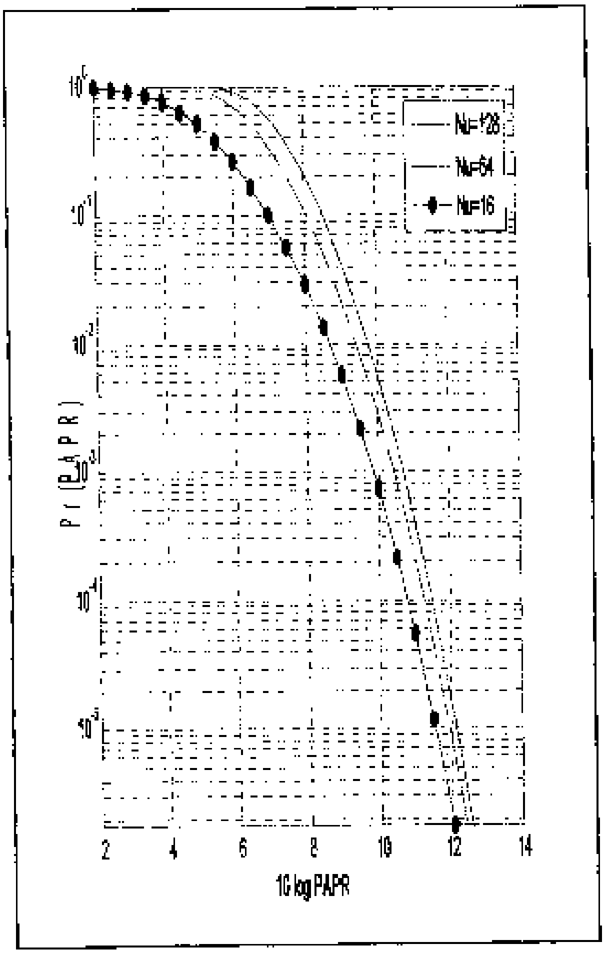

Figure 2: $\Pr(PAPR)$ **for different values of subcarriers.**

It is clearly appeared from the above figure that the values of crest factor occurring with $\Pr(PAPR) \leq 10^{-4}$ are within a range of 1 dB for the values of sub-carriers taken in this simulation. The value of 10^{-4} is taken as a maximum acceptable error value in digital communication systems [1]. A conclusion from this plot is that for a communication system already applies orthogonal frequency division multiplexing as modulation scheme; one should at least use a large number of carriers. Of course, increasing the number of carriers causes other problems like frequency offsets or an increase in time delay especially in the presence of fading conditions. In practice, there will be a trade-off in the design of OFDM systems.

4 Tone Reservations

In 1999 J. Tellado suggests a new technique (as a part of his Ph.D thesis) to reduce the peak to average power ratio of the large sub-carriers OFDM systems [8]. This technique is called tone reservation (TR). The basic idea is to reserve a small set of tones generated separately in the transmitter. These reserved tones are not used for data transmission instead; they are reserved for anti peak signals and they are orthogonal to the other tones which bear data. The goal is to find the time domain signal $c(n)$ that added to the original time domain signal $s_m(n)$ in order to cancel large peaks [8].

Figure 3, shows the operation of TR technique in discrete time domain. When the vector:

$$C(k) = IDFT\{c(n)\} = [C_0, C_1, \ldots, C_{Nt-1}]^T$$

is added to the symbols $X_m(k)$ in the frequency domain, the new time domain signal can be represented as

$$\tilde{s}_m(n) = s_m(n) + c(n)$$
$$= IDFT\{X_m(k) + C(k)\} \qquad (11)$$

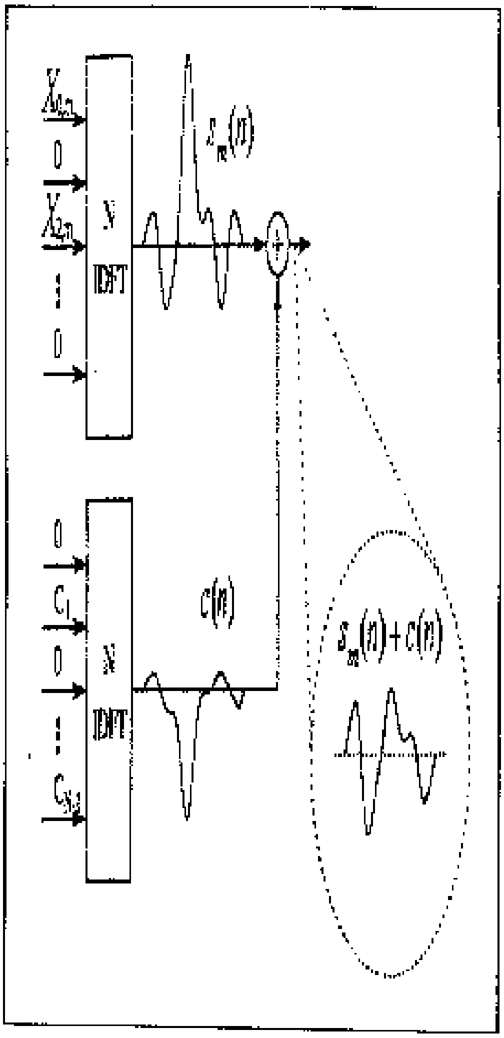

Figure 3: **Operation of the tone reservation Technique.**

The new PAPR of the additive signal $\tilde{s}_m(n)$ is defined as:

$$PAPR = \frac{\max|\tilde{s}_m(n)|^2}{E\left[|s_m(n)|^2\right]}$$

$$= \frac{\max|s_m(n)+c(n)|^2}{E\left[|s_m(n)|^2\right]} \qquad (12)$$

The TR technique restricts the data block $X_m(k)$ and peak reduction vector $C(k)$ to lie in disjoint frequency subspaces i.e. $X_m(k) \cdot C(k) = 0$.

Tellado's original technique reduces peaks in the time domain by iterative subtraction of dirac-like functions and the algorithm to computing $c(n)$ or $C(k)$ and reducing the PAPR of the OFDM signal is presented in [8, 9] and it is summarized in the following section.

4.1 Tellado's Algorithm

This algorithm assumes that a PAPR value less than ζ_0 is wanted and there is a limit for the number of iteration [9].

a. Initial condition: $i = 0$ and $\tilde{s}_m(n) = s_m(n)$.

b. If $PAPR \le \zeta_0$ jump to h.

c.Clip $s_m(n)$ to generate $s_m^{Clip}(n)$.

d. Compute $c(n) = s_m^{Clip}(n) - s_m(n)$.

e. Compute $C(k) = DFT\{c(n)\}$.

f. Compute $\tilde{s}_m(n) = IDFT\{X_m(k) + C(k)\}$.

g. Increment the iteration counter, $i = i + 1$ if $i < Max.Iteration$, jump to b.

h. Transmit $\tilde{s}_m(n)$.

4.2 The Proposed Algorithm

The proposed method is based on two major factors, one deals with the type of searching, the parallel searching is considered in this method, as it can benefit from the linearity of the $IDFT$ to manipulate in parallel the effect of PAPR. The other factor is based on the taken values from the simulation results. Therefore, in this way, we can determine the position of the peaks, from the parallel search and the amount of clipping, from the simulation results.

The following is a summary of such algorithm:

a. Initialize $X(k)$ to be the DFT information vector, with reserved carrier set to zero.

b. Initialize the time-domain vector ($s_m(n)$ *to* $s_m^{(i)}(n)$, $i = 0$), which results from the $IDFT\{X(k)\}$, and calculate the PAPR.

c. Take out the desired peak a_0, from the memory table.

d. Find the values exceeds a_0, from $s_m^{(0)}$ and locations l, if not found jump to h

e. Initialize the vector $C_m^{(0)}$,

$$C_m^{(i)} = \left[\left(\max(0, l_0)\left|s_m^{(i)}\right| - a_0\right)e^{j2\pi\frac{lo}{Nu}} + \ldots, 0,0,0,\right.$$

$$\left.\left(\max(0, l_0)\left|s_m^{(i)}\right| - a_0\right)e^{j2\pi\frac{lo}{Nu}} + \ldots, 0,0,0\right]$$

f. Update the time-domain vector according to

$$s_m^{(i+1)} = s_m^{(i)} - \sum_{l=0}^{2} sign\left(\max(0, l)\left|s_m^{(i)}\right|\right) \cdot IDFT\left\{C_m^{(i)}\right\}$$

g.Initialize the vector $C_m^{(0)}$,

$$C_m^{(i)} = \left[\left(\max(0, l_0)\left|s_m^{(i)}\right| - a_0\right)e^{j2\pi\frac{lo}{Nu}} + \ldots, 0,0,0,\right.$$

$$\left.\left(\max(0, l_0)\left|s_m^{(i)}\right| - a_0\right)e^{j2\pi\frac{lo}{Nu}} + \ldots, 0,0,0\right]$$

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$$s_m^{(i+1)} = s_m^{(i)} - \sum_{l=0}^{2} sign\left(\max(0, l)\left|s_m^{(i)}\right|\right) \cdot IDFT\left\{C_m^{(i)}\right\}$$

i. $i = i + 1$, go to d

j. Transmit $\tilde{s}_m(n)$.

Unlike Tellado's original procedure, the idea of the proposed algorithm is developed from finding that clipped portions of time domain signal which may be written as:-

$$s(n)-s^{clip}(n)=\sum_{i}\beta_i\cdot(\delta\rightarrow l_i) \qquad (13)$$

where the β_i is a value that exceeding the clip threshold $s^{clip}(n)$, the l_i represents the clip location and "$\rightarrow l$" denotes (cyclic) shift by l and δ is the delta dirac function. The dirac function δ requires the whole DFT frame and thus it is not allowed for transmitting any more information. The idea is now to reserve only some of the frequency bins and use them to generate a dirac-like time-domain signal γ that could be subtracted iteratively at peak locations. This can be approximated by:

$$\sum_{i}\beta_i\cdot(\delta\rightarrow l_i)\approx\sum_{i}\beta_i\cdot(\gamma\rightarrow l_i) \qquad (14)$$

Figure (4) shows the dirac-like time domain vector as a result of computer simulation test using the proposed algorithm. According to this algorithm, a percentage of 7% of the total number of sub-carriers are taken with an active value (reserved tons). The locations of these reserved tons are assumed to be at bins (3, 5, 34, 58, 69, 82, 108 and 110). A simple approach to find a suitable dirac-like function γ is to set all reserved carriers (DFT bins) to a constant. When the bins are chosen randomly and after a certain number of iterations, one may be able to find a set of bins that shows a sufficient peak compared to the required vector.

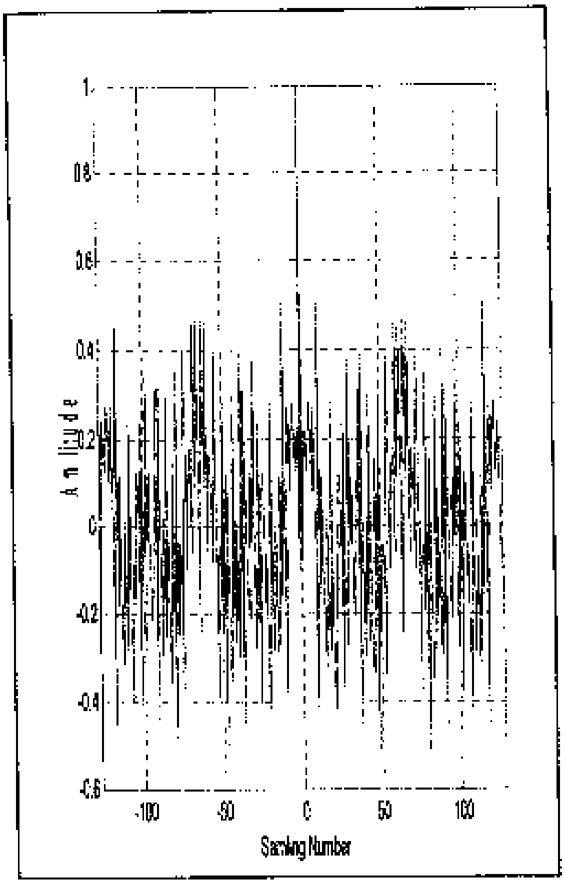

Figure 4: a Dirac-like time-domain vector of the proposed algorithm.

Further tests have been carried out to create the memory data needed in point (c) of the proposed algorithm. Such data is shown in Table (1). This table includes a threshold value for each value of the PAPR of the OFDM system. Sub-carriers of 128 and 7% tone reservation are used in all the following computer simulation tests.

Table 1: Memory data

PAPR	12.5	11.5	10.5
Threshold	0.2739	0.2587	0.2460
PAPR	9.5	8.5	7.5
Threshold	0.2423	0.2422	0.2418

Figure (5) shows a comparison between the probabilities of PAPR of an ideal OFDM system with the proposed system. This figure shows a reduction of 5.25 dB in the PAPR of the system uses the proposed algorithm as compared with the ideal OFDM system. The word ideal means an OFDM system without complexity (without PAPR reduction technique).

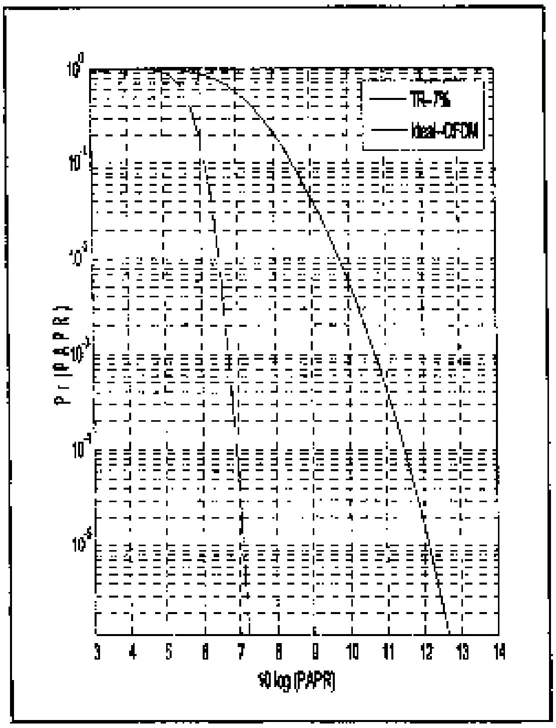

Figure 5: Probability of the PAPR for OFDM systems

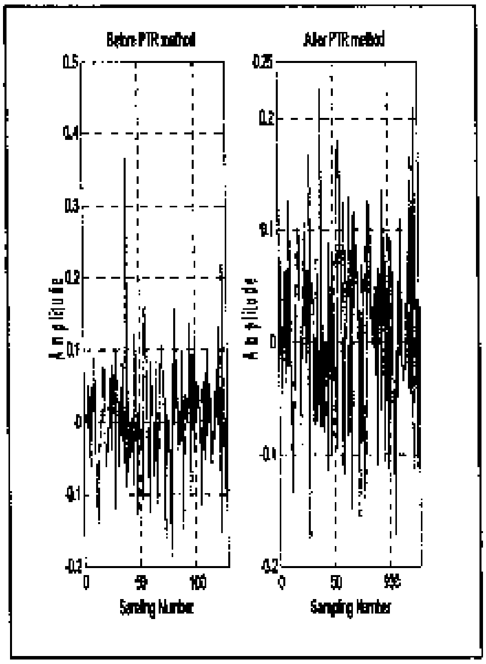

Figure 6: Output OFDM signal before and after applying the proposed TR method.

The reduction of the proposed OFDM signal peaks is clearly appeared in Figure (6).Table 2 shows the difference between the proposed and the conventional tone reservation algorithms in terms of the number of iterations for different values of PAPR.

Table 2: Number of iterations of TR and proposed TR (PTR)

PAPR	12.5	11.5	10.5	9.5
TR	10	11	6	6
PTR	3	4	4	1
PAPR	8.5	7.5	6.5	5.5
TR	28	10	14	14
PTR	6	5	4	4

5 Conclusion

The complexity of the techniques used to reduce the PAPR of the OFDM systems is proportional to the number of sub-carriers. This paper suggests a technique with tone reservation method seems to be one of the best techniques used for large sub-carriers OFDM systems. It allows a good compromise between effectiveness and simplicity, and offers more efficient features. Sometimes, one iteration may be enough to synthesize the reserved signal needed for tone reservation method.

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